Detection of Abnormal Changes in Financial Markets

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Foreign Exchange Market as a complex system
Universal mechanisms of random walks
Detection of abnormal changes
What are financial markets?

“The coming together of buyers (borrowers) and sellers (lenders) to trade financial securities”

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There are more than 1 million markets as a whole in the world.
Foreign exchange market: currency market
Ex. Buy 1 million US-dollar by JP-yen by a rate 123.456 JPY/USD

FX market data is very interesting because:

● Total volume of transaction is the largest in the world

$400 \times 10^{10} \text{ USD (4 Trillion) /day}$

(The whole world trade (materials) is $4 \times 10^{10} \text{ USD /day}$)

● The market is open 24 hours from Monday to Friday

(Continuity of data is better than any other markets)

● Time resolution is very high, 1 msec

● All orders (buy, sell, cancel) are visible

(except user’s names)
From the viewpoint of complex systems

Financial markets are consisted of millions of different components (individuals, companies)

Millions of markets are interacting

Sometimes rules change

Any news can affect the markets

Human desire and emotion also affects (the ultimate intangible complex systems)

Changing every time, no steady state ever existed!

Scientific approaches seem very difficult, but they are extremely important for our society
Good properties for scientific study

The structure of one market is rather simple:
Assembly of buy and sell orders on 1 dimension (price)
(1-dim data with time stamps)

The aim of each dealer can be simple,
buy at a low price and sell at a high price to earn
(Agent-based modeling is available)

We can get high quality data for scientific research
that enable us detail analysis
(Empirical laws are derived from the data)
(Number of direct dealers are limited, about 1000)
Random walk property of market prices

Dollar-Yen rate for 13 years

T. Mizuno, S. Kurihara, M. Takayasu, and H. Takayasu
[Physica A324(2003), 296-302]

For shorter time scale the fractal property breaks down.
Time scale of financial markets

10^{-3} 10^2 10^5 10^7 10^{10}

millisec sec min hours day month year

dealers' strategy
"10-minutes is an epoch"

Algorithm trader's time scale

fractal properties
Financial technology

Langevin model and PUCK model
Why Random Walk?

Buy and Sell orders occur randomly

This is roughly correct, but the mechanism is not that simple

Deeper understanding necessary

A naïve question:
Does the randomness come from outside or is it an intrinsic property of the market? Exogenous? Endogenous?

Exogenous effects exist, of course, also an endogenous mechanism of chaos exists due to the market rule
Every dealer hopes to increase the money by suitable exchange

Common strategies ...

1: Try to buy at a low price and to sell at a high price

2: Follow the trend
**Dealer model** (artificial markets, agent models)

Each dealer has both buying and selling prices

A trade occurs when buying and selling prices meet

\[
\text{Bid}(i) < \text{Ask}(i) \text{ for all } i
\]

\[
\text{Min}\{\text{Ask}(j)\} \leq \text{Max}\{\text{Bid}(i)\}
\]

Buy bids

Sell asks

Trades have attractive interactions for the dealers in the price space

Similar to catalytic reaction

Or matter-antimatter reaction
**Trend-following action**

Appearance of trend \( \cdots \) depends on dealers

A trend continues or turns \( \cdots \) depends on dealers' strategy

We can assume a term proportional to latest price change

\[
d_n \Delta x(t)
\]

Coefficient: positive means trend-follow

Negative means anti-trend action (contrarian)
Compromise:
A seller (or buyer) gradually decreases (or increases) the price

In the simplest deterministic dealer model, we assume that after the deal, the seller becomes a buyer, the buyer becomes a seller.

A hasty dealer has a steeper slope
A slow dealer has a gentle slope
(theses values are given initially)
Evolution of dealers (deterministic, no randomness added)

\[ x_n(t + \Delta t) = x_n(t) + a_n s_n(t) + d_n \Delta x(t) \]

- \( x_n(t) \): n-th dealer’s buying price
- \( a_n \): Trend-follow

(selling price is given by adding the spread L)

A market simulation with 3 deterministic dealers

The mechanism of chaos makes seemingly random fluctuations
What is the chaos in this model?

Chaos is the mechanism of enhancing small difference by nonlinear effect.

Deals are highly nonlinear (step-function type) irreversible interaction.

“Randomness” is produced by this market mechanism intrinsically.

Dealer 0 enhances the small difference by nonlinear effect.

Dealer 1 and 2 are very different.

Dealers are quite similar.

Deals are highly nonlinear (step-function type) irreversible interaction.

"Randomness" is produced by this market mechanism intrinsically.
Without the trend-follow ($d=0$), the market price fluctuates around an equilibrium price.

**Trend-follow ($d\neq 0$) is essential for realization of random walk**

Trend-follow ($d\neq 0$) is also essential for volatility clustering. $\Delta x \equiv x(t+1) - x(t)$

**Volatility clustering occurs automatically by this model**
Price difference distribution follows a power law just like the real market.

\[ P(\geq \Delta X) = \int_{\Delta X}^{\infty} p(\Delta x) d\Delta x \]
\[ \propto \Delta X^{-\alpha} \]


\[ \Delta x \equiv x(t + dt) - x(t) \]

Our latest data analysis clarified that there exist real dealers who behave quite similar to this model, pair-wise ordering (buy-sell) and trend-follow behaviors (preparing a paper)
Theoretically we can show that the motion of mass center is approximated by Langevin eq. in the case d>0

\[ \frac{d^2}{dt^2} \overline{x}(t) = -\mu \frac{d}{dt} \overline{x}(t) + F(t) \quad \overline{x}(t) = \frac{1}{N} \sum_{n} x_n(t) \]

Langevin eq. is the key concept for Brownian motion

Therefore, the dealer model produces price motion similar to Brownian motion

In 1908 Langevin introduced the equation as a model of Brownian motion of a fine particle suspended in water.
Recently, market price dynamics in very short time scale is shown to follow \textit{Langevin equation with non-white noise} by analyzing high quality market data, "order-book data".

\[
\frac{d}{dt} v(t) = -\mu(t)v(t) + G(t)
\]

\textit{Financial Brownian Particle in the Layered Order-Book Fluid and Fluctuation-Dissipation Relations}

Yoshihiro Yura,\textsuperscript{1,4} Hideki Takayasu,\textsuperscript{2,3} Didier Sornette,\textsuperscript{4} and Misako Takayasu\textsuperscript{1,*}
In the case of a colloidal particle motion in water molecules, we expect the following relations

Assume that the colloidal particle is moving to the right. The density of molecules will change around the particle.

In material: Water molecules are invisible, however,
In market: Financial molecules are visible.

Financial Brownian particle
Core size = bid-ask spread
Center at the mid price = (best-bid + best-ask)/2

Inner layer diameter = core size + 2 \( \gamma_c \)
Density change profile when price goes up (right)

Density change

$\gamma$ : the depth from the best bid (ask)

No correlation for $\gamma > 100$

No correlation for $\gamma > 100$

Push by molecule density increase

Pull by molecule density decrease

$\gamma_c = 18$ pips (0.001 yen)

$F_i(t) = \sum_{s:[t,t+\Delta t]} \sum_{x:inner} \left\{ c_x^-(s) - a_x^-(s) \right\} - \left( c_x^+(s) - a_x^+(s) \right)$

Summing up all changes in the inner layer for a time interval

Number of particles increased in the inner layer bid side

Number of particles increased in the inner layer ask side
Velocity of the colloidal particle is approximated by the following quantity

\[ v(t) = \frac{L(\Delta t)}{\Delta t} F_i(t) + \eta(t) \]

- \( F_i(t) \): Layered order flow
- \( L(\Delta t) \): Price change per order change
  \( \rightarrow \) mean free path
- \( \Delta t \): Time window
  100 deals \( \approx \) 160 sec
- \( \eta(t) \): Noise term

The total particle number change in the inner layer (anti-particles are counted with negative sign) is highly correlated with market price velocity.

\[ \text{slope} = \frac{L(\Delta t)}{\Delta t} \]
Layered order flow $F_i(t)$ vs. conventional order flow $F(t)$

$$F(t) = \sum_{x=-\infty}^{\infty} \{ c^-(x,t) - c^+(x,t) - a^-(x,t) + a^+(x,t) \}$$

(sum of increment of all buy orders – sum of increment of all sell orders)

The correlation is much higher than the conventional order flow

The inner layer orders work as driving force accelerating price motion, and the outer layer work oppositely braking the movement
Conventionally, it has been believed that

\[ \text{Price change} \propto \text{demand} - \text{supply} \]

(all buy orders)  (all sell orders)

The fact is

\[ \text{Price change} \propto \text{change of (buy} - \text{sell orders)} \]

only in the inner layer

\[
\begin{align*}
\text{driving force} \\
\text{Outer layer molecules work in an opposite way} \\
\text{drag force (resistive force)}
\end{align*}
\]
Space-time composition of particles

Creation in the inner and outer

- Particle created in the inner layer
- Particle created in the outer layer
- Particle annihilated in the inner layer
- Particle annihilated in the outer layer

Number of particles created in the inner and annihilated in the inner, never being in the outer (driving force)

Number of particles coming from outer and annihilated in the inner (origin of drag force)

Annihilation in the inner, outer
Decomposition of velocity

\[ v(t) = \frac{L(\Delta t)}{\Delta t} F_i(t) + \eta(t) \]

Velocity can be decomposed into \( a_{oi}(t) \) originated and the others.

\[ v(t) = v_{a_{oi}}(t) + v_{others}(t) + \eta(t) \]

Molecules coming from outer to inner

\[ v_{a_{oi}}(t) = \frac{L(\Delta t)}{\Delta t} \sum_{s=0}^{\Delta t} \left\{ -a_{oi}^{-}(t+s) + a_{oi}^{+}(t+s) \right\} \]

\[ v_{others}(t) = \frac{L(\Delta t)}{\Delta t} \sum_{s=0}^{\Delta t} \left\{ c_i^{-}(t+s) - c_i^{+}(t+s) - a_{ii}^{-}(t+s) + a_{ii}^{+}(t+s) \right\} \]

Linear Response relation between \( v_{a_{oi}}(t) \) and \( v(t) \)

\[ v_{a_{oi}}(t) = \int_{-\infty}^{t} \phi(t-s)v(s)ds \]

Response function

\[ |v_{a_{oi}}(\omega)|^2 = |\phi(\omega)|^2 |v(\omega)|^2 \]

Fourier Transform

\[ v_{a_{oi}}(\omega) = \int v_{a_{oi}}(t)e^{-2\pi i\omega t}dt \]
Estimation of the response function

From the data we calculate the power spectrum of $v(t)$ and $v_{aoi}(t)$

\[ \frac{\left\langle \left| \frac{v_{aoi}(\omega)}{v(\omega)} \right|^2 \right\rangle}{\left\langle |v(\omega)|^2 \right\rangle} \]

By taking their ratio, the spectrum of response function is estimated

The functional form of

\[ \left\langle |\phi(\omega)|^2 \right\rangle = \left\langle \frac{\left| v_{aoi}(\omega) \right|^2}{|v(\omega)|^2} \right\rangle \]

is approximated by a Lorentzian.

This means that the response function is an exponential function:

\[ \phi(t) = \phi_0 e^{-\delta t} \]

Average is taken over 200 realizations for window size $2^8$.

$\phi_0 = 0.08, \delta = 0.27$
Derivation of Langevin Equation and resistivity

\[
\begin{aligned}
\mathbf{v}(t) &= \mathbf{v}_{aoi}(t) + \mathbf{v}_{\text{others}}(t) + \mathbf{\eta}(t) \\
\mathbf{v}_{aoi}(t) &= \int_{-\infty}^{t} \phi_0 e^{-\delta(t-s)} \mathbf{v}(s) ds \\
\frac{d}{dt} \mathbf{v}_{aoi}(t) &= -\delta \mathbf{v}_{aoi}(t) + \phi_0 \mathbf{v}(t) \\
&= -\delta \{\mathbf{v}(t) - \mathbf{v}_F - \mathbf{\eta}(t)\} + \phi_0 \mathbf{v}(t)
\end{aligned}
\]

\[
\frac{d}{dt} \mathbf{v}(t) = \frac{d}{dt} \mathbf{v}_{aoi}(t) + \frac{d}{dt} \mathbf{v}_{\text{others}}(t) + \frac{d}{dt} \mathbf{\eta}(t)
= -(\delta - \phi_0) \mathbf{v}(t) + (\delta + \frac{d}{dt}) \{\mathbf{v}_{\text{others}}(t) + \mathbf{\eta}(t)\}
= -\mu \mathbf{v}(t) + G(t)
\]

\(\mathbf{v}_{aoi}(t)\) contributes as the resistivity force

**Drag coefficient (viscosity):**

\[\mu = \delta - \phi_0 = 0.27 - 0.08 = 0.19\]

In the PRL, time is normalized by \(\phi_0\) and

\[\mu' = (\delta - \phi_0) / \phi_0 = 2.2\]
Langevin equation is naturally accepted for colloid in physics

\[ m \frac{d}{dt} v(t) = -\nu v(t) + f(t) \]

- mass
- resistivity
- random force

Conventional finance theory neglects the inertia term

\[ 0 = -\nu v(t) + f(t) \rightarrow v(t) = f(t) / \nu \]

We proved existence of the inertia term for price motion

\[ \frac{d}{dt} v(t) = -\mu(t)v(t) + G(t) \]

\[ \mu(t) = \frac{\nu}{m} \]

Resistivity is time-dependent
Noise term is not white but highly correlated
(to vanish the autocorrelation of v)
Brownian motion of a colloid particle

Water molecules are invisible
“Colloid particle” is observable
From trajectory of a colloid particle
Thermal motion of water molecules are estimated.

Random walk of Market prices
Surrounding molecules are observable
“Particle” is not directly observable
From motion of surrounding molecules the dynamics of “Particle” is estimated

A lot of similarities are confirmed

The biggest difference is that
A financial molecule costs 1 million USD!
Langevin equation with random resistivity produces power law distributions (known as Kestin-process)

\[
\frac{d}{dt} v(t) = -\mu(t)v(t) + G(t) \\
v(t + \Delta t) = (1 - \mu(t)\Delta t)v(t) + f(t)
\]

\[v(t + \Delta t) = b(t)v(t) + f(t)\]

\[P(\geq |v|) \propto |v|^{-\alpha} \quad < b(t)^{\alpha} >= 1\]

Power law distributions of financial markets can be explained by temporal fluctuation of resistivity of markets

Volume 79, Number 6  Physical Review Letters  11 August 1997

Stable Infinite Variance Fluctuations in Randomly Amplified Langevin Systems

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Our data analyses and models of financial markets

**Order-Book analysis** 2014-2015
Statistical properties of OB and correlation between changes in number of order-book and velocity

**Layered OB model** 2014-2015
Slow response of outer-layer contributes as "viscosity"

**Time dependent Langevin eq.** 1997-2014
New Physics

**Deviations from RW** 2002-2014

**Trend-follow Potential force**

**ARC • GARCH model** 2005-2012
Viewing only variance

**ARCH • GARCH model** 2005-2012
Practical application

**Crash alarm** 2009-2013
Macro limit 2009-2013

**Inflation Eq.** 2009-2013
Macro Economy

**Artificial Markets** 1992-2014

**Dealer model**

**Time series analysis** 2012-2015
Change-point detection 2013-2015

**Knudsen number of markets** 2015-
PUCK model bridges physics and finance

**Potential of Unbalanced Complex Kinetics**

\[
v(t) = x(t + 1) - x(t) = -\frac{b(t)}{M-1} \left( x(t) - x_M(t) \right) + f(t)
\]

\[
= -\frac{\partial}{\partial x} \left\{ \frac{b(t)}{2(M-1)} \left( x(t) - x_M(t) \right)^2 \right\} + f(t)
\]

The potential

\[
x_M(t) \equiv \frac{1}{M} \sum_{k=0}^{M-1} x(t-k)
\]

: Moving average of market price

\[
x(t + 1) - x(t)
\]

Moving average of price

Misako Takayasu et al
Physica A370 (2006), 91
PRE 79(2009), 051120
PRE 80 (2009), 056110

Non-trivial relation
Random walk
An example of PUCK analysis for a typical day

Price chart

Market Potential

Potential coefficient

$b > 0$

stable

Unstable
Microscopic anomalies are reproduced by PUCK model

- Time series
- Diffusion
- Auto-correlation
- PUCK potential

Real data

PUCK model using measured $\{b(t)\}$ with random noise $f(t)$
Market potential force in the dealer model

\[ p_i(t + \Delta t) = p_i(t) + d < \Delta P >_M + c \cdot f_i(t) \]

**Prediction term**

- **Contrarian** \((d<0)\)
- **Stable potential** \((b>0)\)
- **Unstable potential** \((b<0)\)

Dealers’ strategy is gradually changed from contrarian to trend-follower

Trend-follower \((d>0)\)

The first attack

The second attack

Pentagon attacked

A WTC collapsed

Market potential changed drastically by the news

The market was stable. It became very unstable. It gradually stabilized.
Observation of asymmetric potentials before large crash (1998 for a month) Yen-Dollar

\[ P(t+1) - P(t) = \sum_{k=1}^{3} b_k(t) \cdot [P(t) - P_M(t)]^k + f(t) \]

Stable attractive potential
Asymmetric potential: symptom of a crash
Asymmetric potential: onset of crash
Unstable but symmetric random walk

Physical Review E 80 (2009), 074911
PUCK model can be used by financial practitioners via Bloomberg App Portal: “Order Book Tracer”
Derivation of time evolution of variance by PUCK model

\[ x(t + \Delta t) = x(t) - \frac{b(t)}{2} \{x(t) - x(t - \Delta t)\} + f(t) \]

\[ M = 2 \]

In the continuum limit, PUCK model becomes Langevin Eq.

\[ \frac{d}{dt} v(t) = -\mu(t)v(t) + F(t) \]

1. The mass is normalized to be unity
2. Viscosity changes with time

\[ v(t) \equiv \frac{x(t) - x(t - \Delta t)}{\Delta t} \quad \mu(t) \equiv \frac{1 + b(t) / 2}{\Delta t} \quad F(t) \equiv \frac{f(t)}{\Delta t} \]

\[ \frac{\partial}{\partial t} w(v, t) + \frac{\partial}{\partial v} \{ -\mu(t)vw(v, t) \} - D_v \frac{\partial^2}{\partial v^2} w(v, t) = 0 \]

\[ w(v, t) \] : Probability density of velocity

Fokker-Planck equation holds for the probability density of velocity

\[ \frac{d}{dt} \sigma(t)^2 = -2\mu(t)\sigma(t)^2 + 2D_v \]

\[ \sigma(t)^2 = \int v(t)^2 w(t) dt \]

\[ 2D_v \delta(t - t') \equiv \langle f(t)f(t') \rangle \]

Time evolution of variance

\[ \sigma(t + \Delta t)^2 = 2D_v \Delta t + \{1 - 2\mu(t)\Delta t\} \sigma(t)^2 \]

ARCH-GARCH model
By applying the Particle filter version of PUCK model, PUCK parameters, $b(t)$ (the potential coefficient) and $M$ (the size of moving average) are estimated in very short time

Y.Yura, H.Takayasu, K.Nakamura, M.Takayasu

Particle filter method:
$10^4$ PUCK models with different parameters are run in parallel, and they evolve like species of life

A test: sudden change of parameters

Particle filter version

Conventional method

Particle filter PUCK is suitable for real time data analysis
In this example the change of market condition is detected within 10 seconds.
Example: USD-JPY for a week (Mar 2011)

Flash Crash

zooming

1 hour

Flash Crash takes more than 10 minutes continuous unstable states with exponential growth

If the market is too much unstable, the viscosity becomes negative, and price difference grow exponentially!

\[
\frac{d}{dt} v(t) = +\mu \cdot v(t) + G(t) \quad v(t) \propto \exp(\mu \cdot t)
\]
Non-stationary time series analysis • change-point detection
11 years daily data of Australia Dollar / Japanese Yen

By human eye we may segment the time series

There are many methods for change point detection, here, we introduce a most basic non-parametric method (not assuming specific functional form of statistical model).

We apply Fisher’s exact probability test for the change-point detection. Non-stationary time series is segmented into stationary periods by the p-value (the probability that steady independent random fluctuation can realize the bias) (Fisher's test is very popular in medicine).

\[
p = \frac{(a + b)!(c + d)!(a + c)!(b + d)!}{n!a!b!c!d!}
\]
Assuming that the whole interval is statistically stationary, then this level of deviation \((a,b,c,d)\) occurs with probability less than \(10^{-25}\).

We consider that the minimum point is the strongest change-point.

Fig. 4 Minimum p-values for each horizontal threshold.
The time series is segmented automatically by the statistical analysis
For p-value smaller than $10^{-4}$

Price difference

$p=3 \times 10^{-27}$

$p=3 \times 10^{-8}$

$p=10^{-10}$

$p=6 \times 10^{-5}$

$p=9 \times 10^{-9}$
Spending about 25 years, we are ready for the next step

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- 2014-
  - Statistical properties of OB and correlation between changes in number of order-book and velocity

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  - Knudsen number of markets 2015-

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- 2002-
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- 1992-

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- Financial Technology
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**Macro Technology**

**Practical application**

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“Observing the markets as it is” is the first step for establishment of science of financial markets